

Double-Slit Interferometry with a Bose-Einstein Condensate

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A Bose-Einstein “double-slit” interferometer has been recently realized experimentally by Y. Shin et. al., *Phys. Rev. Lett.* **92** 50405 (2004). We analyze the interferometric steps by solving numerically the time-dependent Gross-Pitaevski equation in three-dimensional space. We focus on the adiabaticity time scales of the problem and on the creation of spurious collective excitations as a possible source of the strong dephasing observed experimentally. The role of quantum fluctuations is discussed.

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Introduction. Several current efforts in the field of dilute Bose-Einstein condensates (BEC) are focusing on the creation of new technological devices, including quantum computers [1] and ultrasensitive interferometers [2] to detect and measure weak forces. Atom wave interferometry already provides unprecedented sensitivities to detect rotations, accelerations, and gravity gradients [3]. Performances can be further improved with interferometers based on BEC, which are the highest brilliant coherent sources of matter waves and which allow for larger separations between different interferometric paths.

Interference between two spatially separated condensates was first demonstrated in [5], and theoretically analyzed in [7]. A BEC trapped in a harmonic magnetic trap was split in two symmetric halves by a laser knife. After releasing the external fields, the interference pattern of the overlapping condensates was observed with destructive imaging. The measured relative phase, however, was not reproducible from shot to shot but was randomly distributed due to the presence of a large noise in the relative positions of the laser and the magnetic trap and of parasite currents when switching off the magnetic fields. Reproducible interference patterns were eventually observed by trapping a condensate in deep optical periodic potentials [6]. Neighboring wells of optical lattices, however, are separated only by a fraction of a micron and cannot be individually addressed, limiting their applications in technological devices.

Recently, a stable double-well trap has been created in [4]. A single collimated laser beam was split with an acoustic-optical modulator, and finally focused by a lens. A single, cigar-shaped condensate was trapped in a double-well having a barrier much smaller than the BEC chemical potential, see Fig. (1a). The condensate was then split along the axial direction by linearly increasing, in a ramping time t_{ramp} , both the distance between the two wells and the height of the interwell barrier, see Fig. (1b). The final distance between the two condensates was $\sim 12 \mu\text{m}$, allowing for *individual* addressing and manipulation. After holding the two condensates in the respective traps for a time t_{hold} , the confining field was turned off. The interference pattern of the two overlapping condensates was measured by destructive tech-

niques and was reproducible in different realizations of the experiment. However, a loss of coherence was observed when the condensates were held in the separated wells longer than $t_{hold} \approx 5$ ms. This strong dephasing has been tentatively attributed [4] to axial and breathing mode excitations created during the splitting of the condensate. Attempts to increase the adiabaticity of the process with larger separation times t_{ramp} did not improve the stability of the measured phase. The dephasing manifested itself as a gradual decrease of the phase contrast accompanied by a bending and kinking of the interference pattern, which eventually makes the phase measurement impossible.

In this Letter we theoretically analyze the MIT experiment as a prototype of a general BEC interferometer. Our analysis is quite general, and is relevant, for instance, in the study of the splitting and recombination of BEC propagating in atom-chip wave guides [8]. We focus on the role of the collective excitations created during the splitting process as a possible dephasing mechanism. We numerically solve the full three-dimensional time-dependent Gross-Pitaevskii (GP) equation to reproduce the interferometric steps realized in the experiment. We study the adiabaticity of the splitting process and predict the interference contrast as a function of the various time scales of the problem. We finally consider the corrections to the GP dynamics arising from quantum fluctuations.

Protocol. Our simulation of the interferometric experiment consists of four steps: i) *Initialization*. We load a BEC into an optical double well potential of gaussian shape in the x direction and of harmonic shape in the y and z directions with frequencies ω_y and ω_z , respectively. We fix an initial separation x_0 between the wells in such a way that the height of the potential barrier is much smaller than the chemical potential, Fig. (1a). We solve the GP equation in imaginary time in order to find the ground state (GS) of the system. ii) *Separation*. We separate the wells by ramping linearly from x_0 at time $t = 0$ to x_{ramp} at t_{ramp} , Fig. (1b). iii) *Holding*. Once the wells are separated, we generally allow the wavefunction to evolve in the trap for a time t_{hold} . iv) *Ballistic (free) propagation*. After a time $t = t_{ramp} + t_{hold}$, we sim-

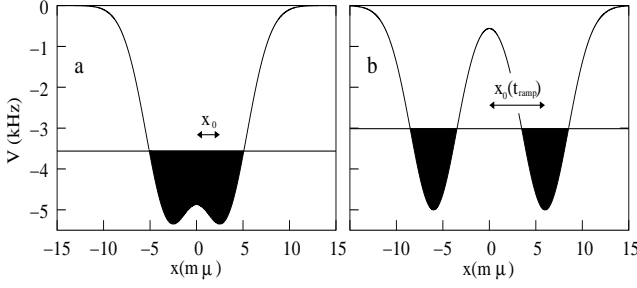


FIG. 1: Double Well potential in the x direction. a) The initial condensate with a distance between the two wells of $x_0 = 3 \mu\text{m}$ and the chemical potential higher than the barrier height. b) Displaced wells in a time $t = t_{\text{ramp}}$ with $x_0(t_{\text{ramp}}) = 6 \mu\text{m}$. In this configuration we have two independent condensates.

ply release the trapping potential and allow the packets from the separated wells to merge and overlap, generating an interference pattern. The condensates are finally imaged after a time t_{free} of ballistic expansion by integrating along the y direction, simulating in this way the data collected experimentally. We cast the interference problem into a three-dimensional Gross-Pitaevskii (GP) equation. We introduce scaled units with the energy in $\hbar\omega_z$, length in $d_z = \sqrt{\frac{\hbar}{m\omega_z}}$, and time in ω_z^{-1} , obtaining

$$i\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \left[-\frac{1}{2}\nabla^2 + V_{\text{ext}}(\mathbf{r},t) + V_{\text{NL}}(\mathbf{r},t)\right]\psi(\mathbf{r},t), \quad (1)$$

where $V_{\text{NL}}(\mathbf{r},t) = 4\pi N \frac{a}{d_z} |\psi(\mathbf{r},t)|^2$ is the nonlinear potential arising from the interatomic interaction, with a the scattering length and N the number of condensate atoms in the trap. The external potential $V_{\text{ext}}(\mathbf{r},t)$ is given by a combination of a harmonic confinement along the y, z -directions, $V_{ho}(y,z) = \frac{1}{2}\left(\frac{\omega_y^2}{\omega_z^2}y^2 + z^2\right)$, and a double-well time-dependent gaussian confinement along the x direction $V_{dw}(x,t) = -\frac{V_0}{\hbar\omega_z} \left[e^{-(x-x_0(t))^2/2\sigma^2} + e^{-(x+x_0(t))^2/2\sigma^2} \right]$, where $2x_0(t)$ is the distance of the potential wells at time t , and σ is the width of each gaussian well. We base our simulations on the MIT experiment [4] considering $N \sim 510^5$ ^{23}Na atoms. The interatomic scattering length is $a = 2.8 \text{ nm}$, and the trap parameters are $\omega_y = 615 \times 2\pi \text{ Hz}$, $\omega_z = 30 \times 2\pi \text{ Hz}$ (giving a length unit $d_z = 3.8 \mu\text{m}$), $V_0 = \hbar \times 5 \text{ kHz}$, and $\sigma = 2.5 \mu\text{m}$. The bottom portion of each gaussian well approximates a harmonic potential with frequency $\omega_x^h = \sqrt{\frac{V_0}{m\sigma^2}} = 2\pi \times 593 \text{ Hz}$. In the initial configuration, Fig.(1,a), $x_0 = 3 \mu\text{m}$ and the chemical potential $\mu = \hbar \times 1.78 \text{ kHz}$. The wells are then separated to a distance $x_0 = 6 \mu\text{m}$, Fig. (1,b), creating two condensates each with chemical potential $\mu = \hbar \times 2.04 \text{ kHz}$.

Results and Discussion. We performed GP simulations by varying the parameters t_{ramp} and t_{hold} keeping fixed the expansion time $t_{\text{free}} = 5.3 \text{ ms}$ [10]. The cen-

tral goal of this section is to extract the relevant adiabaticity time scale of the double-slit BEC interferometer. This requires studying the excitations created during the splitting of the trap and how these propagate during the holding time.

Ramp time. We first examine the behavior of the system as a function of the separation time, t_{ramp} , for fixed $t_{\text{hold}} = 10.6 \text{ ms}$. In Fig. (2), we show the contours of the y -integrated probability density $P_y(x,z,t)$, which closely resemble the projected images of the experiment. For short separation times we have observed considerable distortions and dephasing in the interference patterns.

Given the symmetries of the system, low-lying excitations can only have even parity. For example, in the axial direction the longest oscillation period is $\tau_z \sim \frac{\pi}{\omega_z} = 16.6 \text{ ms}$, and faster ramping times can easily excite large amplitude monopole and quadrupole oscillations. However, we have numerically checked that such collective modes retain a high degree of collectivity (due to the harmonicity of the trap) and cannot be a source of dephasing. Nonlinear couplings, which can potentially destroy the interference patterns, apparently occur at much longer times than those considered in the experiment.

We now consider the frequencies of low-lying even excitations along the radial direction as a function of the interwell distances. The oscillation periods first increase along with the increase of the interwell barrier height. These reach a maximum value when the difference between the energy of the interwell barrier and the chemical potential becomes equal to the frequency of the first odd collective mode, corresponding to the Josephson “plasma” oscillation. Further increasing the interwell distance results in the periods decreasing to a final saturation once the two wells are completely separated. This behaviour reflects the evolution of the single particle energies of the double well during the ramping process. The maximum oscillation period sets the *relevant* adiabaticity time scale of the double-slit BEC interferometer.

In each separate well, both even and odd excitations modes can be excited along the radial axis. The lowest in energy is the dipole collective mode, whose oscillation frequency can be calculated in a variational approach to be $\omega_D^2 = \omega_x^h^2 \left(\frac{1}{1+\gamma^2/\sigma^2}\right)^{(3/2)}$, with γ the radial width of the condensate. The trapping potential is highly anharmonic and the corresponding oscillations frequencies strongly depend on the chemical potential of the system, even in the Thomas-Fermi limit (contrary to what happens with an harmonic confinement). With $N = 10^5$, the largest frequency is obtained at an interwell distances $x_0 \sim 3.7 \mu\text{m}$ and chemical potential $\mu \simeq 0.64 V_{dw}(x=0)$: $\omega_{\min} = 0.65 \omega_x^h$, and the oscillation period is $\tau_{\max} = 1.54 \tau_x^h$. For comparison, when $x_0 = 6 \mu\text{m}$, and the two wells are completely separated, $\omega_D = 0.75 \omega_x^h$ and the oscillation period is $\tau_D = 1.33 \tau_x^h = 2.58 \text{ ms}$.

The breaking down of adiabaticity, however, is not the only source of dephasing. We have not observed any loss of visibility after imposing harmonic confinement along

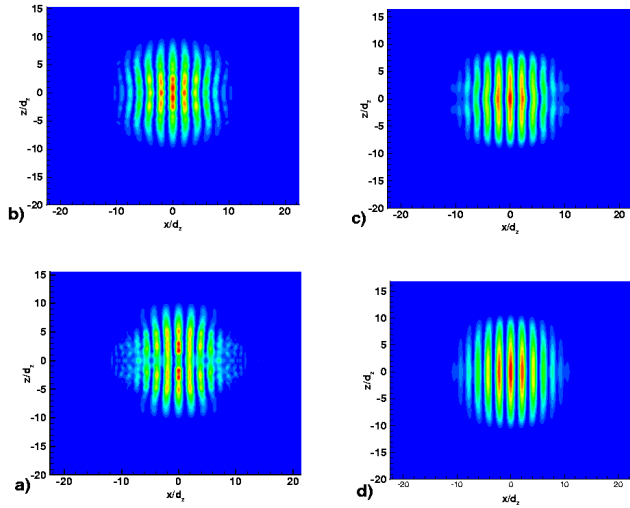


FIG. 2: Contour plots of the probability density integrated over the y -coordinate $P_y(x, z, t)$ from 3D simulations for $t_{hold} = 10.6$ ms and $t_{free} = 5.3$ ms, as a function of the ramping time t_{ramp} : a) 2.7 ms, b) 5.3 ms, c) 10.6 ms, and d) 21.2 ms.

the x axis. On the other hand, we have found that, in the presence of the gaussian confinement, the interference pattern quickly degrades while increasing the number of atoms, keeping all other parameters fixed. Fast rampings induce large amplitude oscillations along the radial direction of each trap. Due to the combined anharmonicity of the confining potential and the non-linearity arising from the interatomic interaction, the energy of the excitation quickly redistributes among different modes, rapidly damping the collective oscillation. We have not observed any coupling of such modes with oscillations among the radial axis. It is useful to recall that the total energy of the system remains conserved in the GP simulation, and the effective damping is only due to redistribution of the collective energy among different Bogoliubov excitations levels. With smaller oscillation amplitudes, induced with longer ramping times, the visibility of the fringes has been greatly improved Fig.(2). However, this effect is in contrast with the experimental results [4] where an increasing of the ramping time, even well beyond τ_{max} , does not improve the interference pattern, which degrades for $t_{hold} \geq 5$ ms (see Fig. 3 in [4]), independently of t_{ramp} .

Hold Time. We now investigate the role of the hold times t_{hold} in the deterioration of the interference pattern. Fig.(3) presents contours for the y -integrated probability density for different t_{hold} and fixed $t_{ramp} = 5.3$ ms. We clearly note a growing deformity of the patterns as the confinement time increases, along with the redistribution of the initial energy among various modes. The maximum dephasing indeed occurs when the oscillation is completely damped. Our simulation goes in the direction

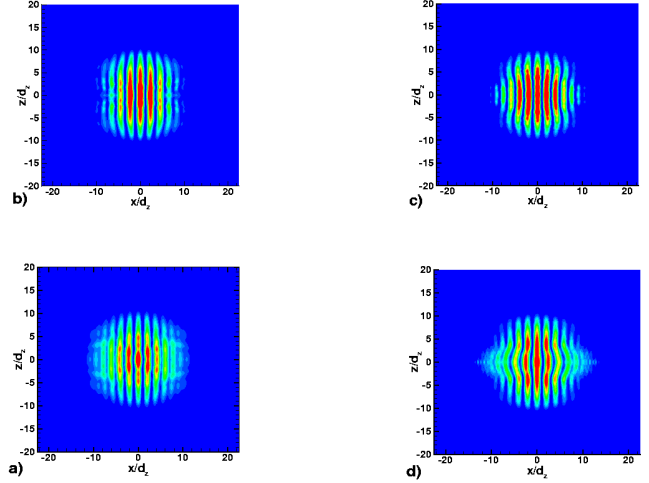


FIG. 3: Contour plots of the probability density integrated over the y -coordinate $P_y(x, z, t)$ for $t_{ramp} = 5.3$ ms and $t_{free} = 5.3$ ms as a function of the hold time t_{hold} : a) 0, b) 5.3 ms, c) 10.6 ms, and d) 21.2 ms.

of the experiment: an increasing of t_{hold} corresponds to a growing distortion of the fringes pattern. However, there is an important difference. We do not observe a complete distortion of the fringes pattern for $t_{hold} \geq 5$ ms, as in [4]. Instead, we find distorted but still defined fringes even for $t_{hold} \approx 20$ ms. Moreover, such distortion can eventually be reduced by increasing the ramping time, as suggested by Fig.(2). Changing the relative depth of the two wells to include an asymmetry in the x -direction, has not lead to a more rapid deterioration of the fringe contrast.

Another important difference between the experimental findings and our GP simulations occurs in the shape of the distorted interference pattern. We always find symmetric bending or kinking of the fringes, contrary to that observed in Fig. (3) of [4]. The asymmetric kinking of the experiment suggest the presence of slight additional geometry variations of the two traps. In the real experiment, indeed, the shape as well as the parameters of the trap can be slightly different from the corresponding mathematical form we have used in our simulations. A small disalignement of the two traps, for instance, can induce small amplitude out of phase dipole oscillations along the z axis which can originate an asymmetric bending. Residual excitations due to the transportation of the condensate to the “science chamber” [4] prior the initialization step i), might also be the source of the dephasing observed experimentally. [12]. We can conclude this part by stating that the excitations coming from the creation of two separated condensates give significant distortion and loss of contrast in the interference fringes, but they do not destroy completely the pattern. In general such distortions increase with t_{hold} , as in the experiment, and with the chemical potential, but it can be substantially

reduced by decreasing the ramping time t_{ramp} .

Beyond Gross-Pitaevskii. The Gross-Pitaevskii framework does not account for dephasing arising from quantum (many-body) correlations. While, on the one hand, a full calculation of the quantum dynamics is well beyond current computational capabilities, simple estimates [13, 14, 15, 16] of the dynamical evolution of the fringe contrast can be carried out within the two-mode approximation and tested experimentally. The quantum phase Hamiltonian governing the condensate is (setting $\hbar = 1$) [11]:

$$i\frac{\partial\Psi}{\partial t} = \left[-\frac{E_c}{2}\frac{\partial^2}{\partial\phi^2} - E_j \cos\phi - \frac{E_j^2}{N^2 E_c} \cos 2\phi \right] \Psi, \quad (2)$$

where $\Psi(\phi, t)$ is the probability amplitude for the relative phase ϕ of the two condensates. The “charging” energy $E_c = 2\frac{\partial\mu}{\partial N}$ remains approximately constant during the splitting of the condensate, while the Josephson coupling energy E_j decreases exponentially with the interwell distance. In the strong coupling limit $E_j/E_c \gg 1$ (achieved when the chemical potential is close enough to the interwell barrier), the phase oscillation frequency is $\omega_j = \sqrt{E_c E_j + 4E_j^2/N^2}$. The phase probability $|\Psi(\phi, t)|^2$ has a gaussian distribution with dispersion $\sigma^2 = \frac{1}{2}\frac{E_c}{\omega_j} \ll 1$. As long as $\sigma(t)$ remains small during the dynamics, the system can be described in the GP framework. With a linear ramping, $d(t) = 2x_0 t/\tau_{ramp}$, and in the *WKB* approximation, the Josephson coupling energy is $E_j(t) \sim e^{(-t/\tau_{eff})}$, with $\tau_{eff} = \tau_{ramp}/S$ and effective action $S = 2x_0\sqrt{2m(V_0 - \mu)}$. At a first stage, while E_j decreases with time, the amplitude $\Psi(\phi, t)$ follows adiabatically the ground state of the effective quantum potential in Eq.(2). Breakdown of adiabaticity occurs at the freeze-out time t_f given by $\omega_j(t_f) \simeq \frac{2\pi}{\tau_{eff}}$, namely when the Josephson period becomes equal to the characteristic time of the change of the Josephson coupling energy $\tau_{eff} = -(\frac{dE_j}{dt}/E_j)^{-1}$. At longer times the dynamics is dominated by the kinetic part of the quantum Hamiltonian Eq.(2). The temporal evolution of the phase fluctuations is therefore given by $\sigma^2(t) = \sigma^2(t_f) + E_c^2/4\sigma^2(t_f)t^2$, with $\sigma(t_f)$ being the width calculated at the freeze-out time. Replacing the corresponding expressions, we finally obtain:

$$\sigma^2(t) = \frac{E_c}{2}(\omega_j(t_f)^{-1} + \omega_j(t_f)t^2) \simeq \frac{E_c}{2}\left(\frac{\tau_{ramp}}{2\pi S} + 2\pi S \frac{t_{hold}^2}{\tau_{ramp}}\right), \quad (3)$$

which is the central result of this section. To experimentally test the quantum phase dynamics Eq.(3) it would be necessary to average over several identical interferometric realizations. In each experiment, an interference pattern will actually be observed [17], but with a relative phase chosen randomly with a gaussian distribution of width given by Eq.(3). The quantum dynamics will therefore manifest itself as a loss of fringe visibility of

the ensemble averaged interference patterns, which will be almost completely washed out when $\sigma \sim 2$. Replacing the experimental values of [4], this happens with a holding time $t_{hold} \simeq \sqrt{\frac{4\hbar^2}{\pi S E_c} \tau_{ramp}} \simeq 2\pi 50$ ms.

Conclusions. We have theoretically studied the double-slit interferometer recently created in [4] solving numerically the full 3D time-dependent Gross-Pitaevskii equation. We have studied the adiabaticity time scales of the system, concluding that the dephasing arising from the creation of spurious excitations can be strongly reduced by increasing the ramping time of the double-well. Such findings indicate that the loss of visibility and the bending of the interference pattern observed experimentally should arise from a different noise source.

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